Astrophysics to z~10 with Gravitational Waves

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Abstract

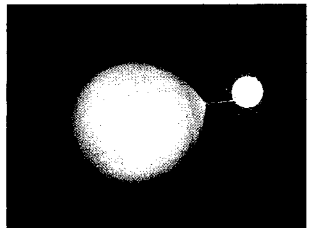
The most useful characterization of a gravitational wave detector's performance is the accuracy with which astrophysical parameters of potential gravitational wave sources can be estimated. One of the most important source types for the Laser Interferometer Space Antenna (LISA) is inspiraling binaries of black holes. LISA can measure mass and spin to better than 1% for a wide range of masses, even out to high redshifts. The most difficult parameter to estimate accurately is almost always luminosity distance. Nonetheless, LISA can measure luminosity distance of intermediate-mass black hole binary systems (total mass~10^4 $M_\odot$) out to z~10 with distance accuracies approaching 25% in many cases. With this performance, LISA will be able to follow the merger history of black holes from the earliest mergers of proto-galaxies to the present. LISA's performance as a function of mass from 1 to 10^7 $M_\odot$ and of redshift out to z~30 will be described. The re-formulation of LISA's science requirements based on an instrument sensitivity model and parameter estimation will be described.

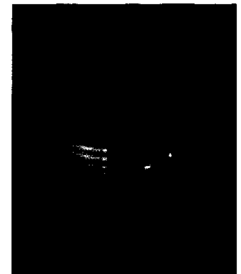
The Problem

- ***Gravitational wave detection is "capability-driven," meaning that the gravitational wave science that we want to do is determined by the measurement capability that we can muster.***
- ***However, circumstances compel us to talk about the science that we can perform to justify.***
- ***NASA requires that the instrument performance requirements be strictly derived from the science requirements.***
- ***How it actually happened:***
  - *Conceive an instrument that seems promising*
  - *Assess what the instrument can do*
  - *Figure out what sources can be observed with that capability*
  - *Figure out what science one can do with those observations*
  - *Iterate, endlessly.*
- ***So, what can one do with a GW detector, such as LISA?***

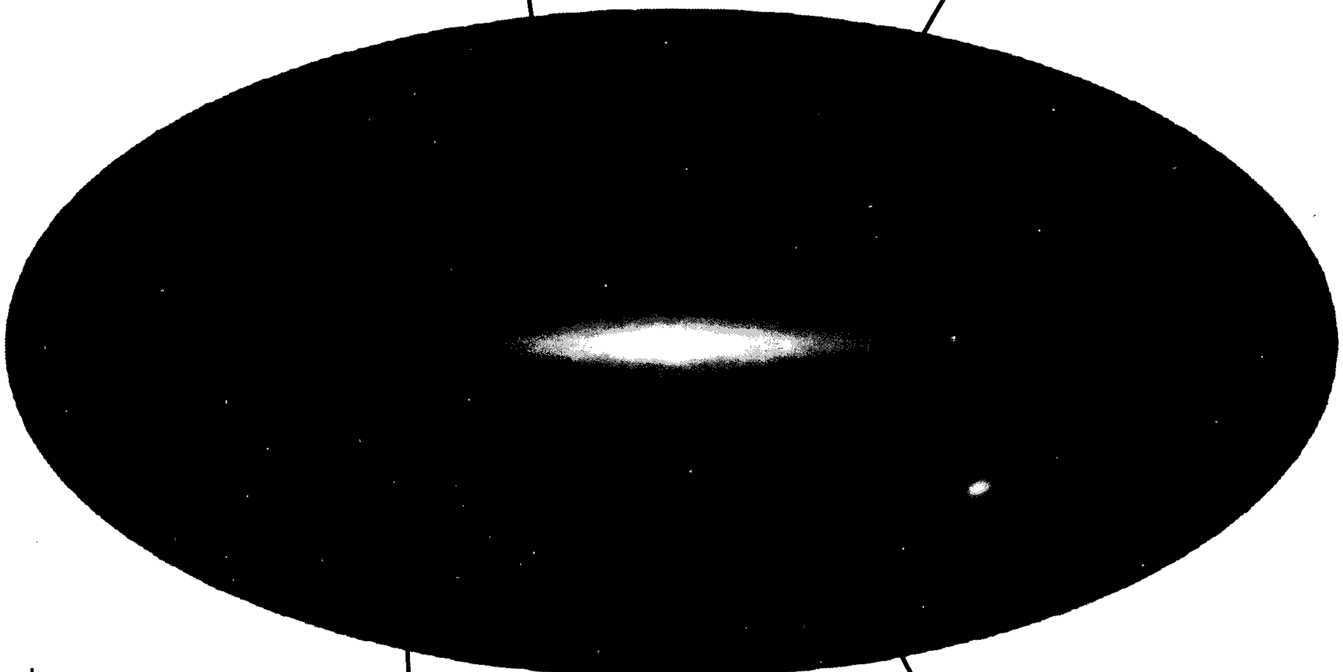
The LISA Sources

Ultra-compact binaries
- $\sim 1\ M_\odot$
- Galactic and extragalactic
- 1000's - 10,000
- Confusion foreground

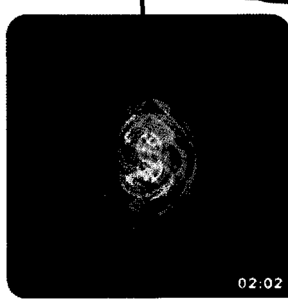




Extreme mass-ratio inspirals
- $\sim 10/\ 10^6\ M_\odot$
- $z < 1$
- 10's - 100 per year



Massive and intermediate-mass black hole binaries
- 10^2 - $10^7\ M_\odot$
- $z < 20$
- 10's to 100 per year

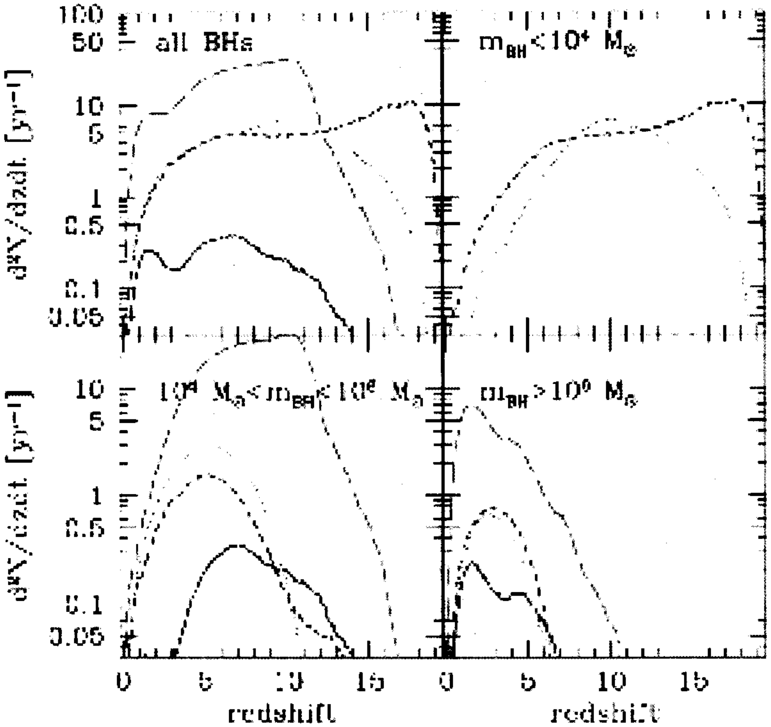


Cosmological back-grounds, bursts and unforeseen sources

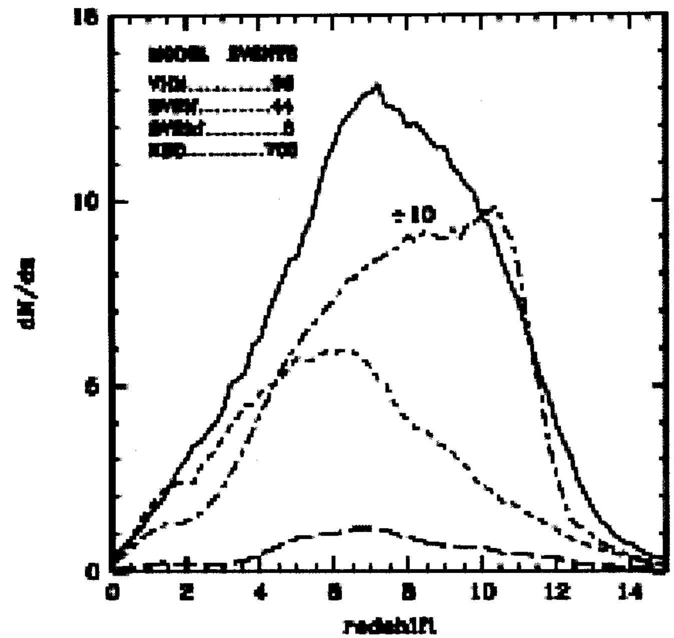
GW Observations

- ***Concentrate on inspirals, rather than burst, quasi-periodic, or stochastic sources.***
- ***With few exceptions, we don't know the sources beforehand***
  - *We only know the general form (template) of the signals to look for.*
  - *The parameters of specific sources are unknown prior to observation, and other characteristics, such as the event rate, are poorly known.*
  - *The properties of the sources are determined in the process of "detection" through template matching.*
- ***LISA can determine source parameters pretty well (by astrophysics standards), especially quantities encoded in frequency.***
- ***To a significant degree, we're going to discover what science can be done with our GW detection as we go along.***
- ***So, how do we characterize the discovery space?***
- ***How do we show what sources can be observed, should they exist?***

Merger Trees



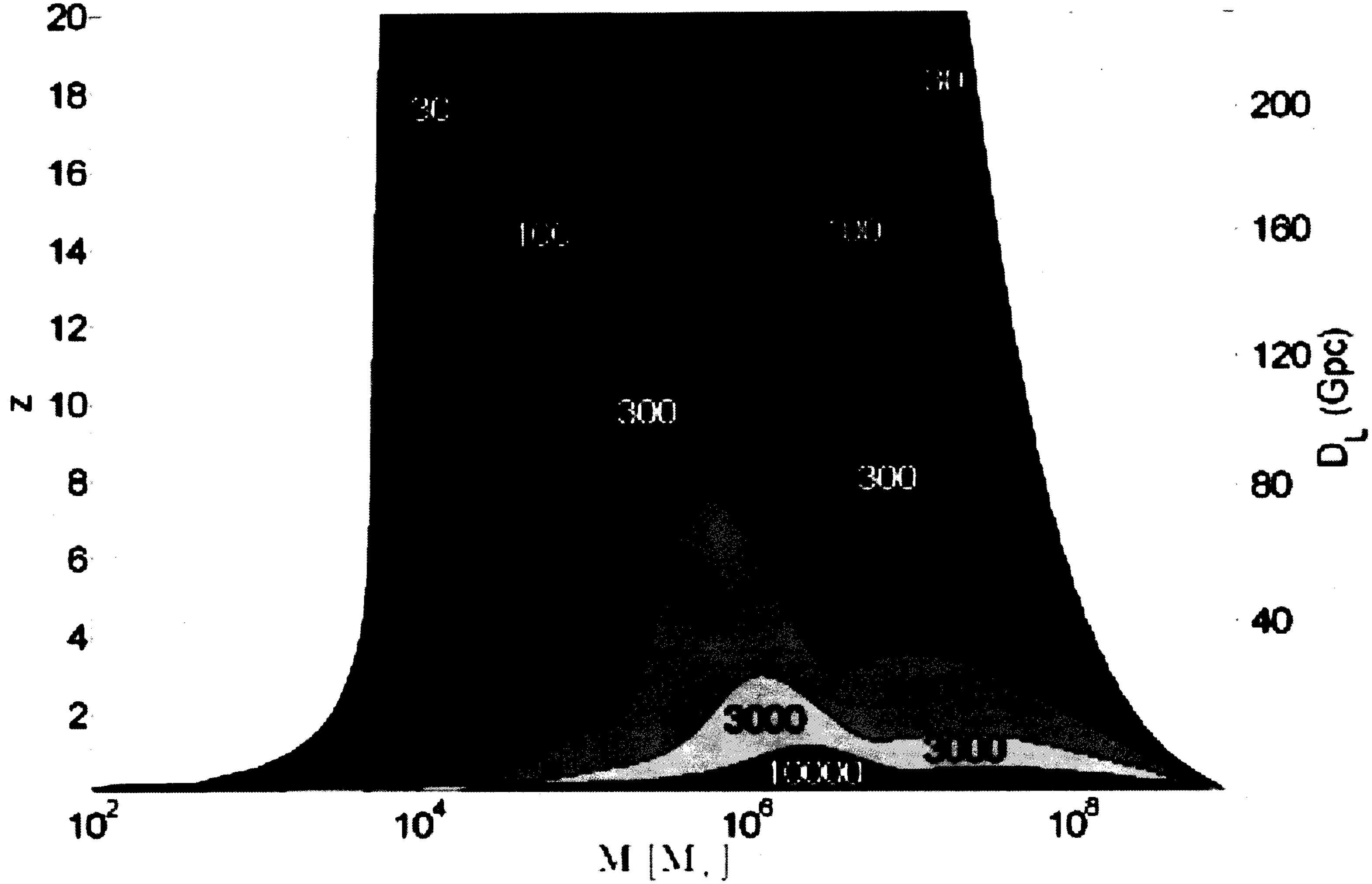
Sesana et al 2004 ApJ, 611, 623



"Detection" vs. "Observation"

- ***"Detection" means identifying the presence of a source***
  - *Determine the likelihood of a source signal in the data*
  - *Figure of merit is SNR*
  - *Detection can be useful, but doesn't answer specific scientific questions, like what is the merger rate of intermediate mass black holes at z=10?*
- ***"Observation" means getting useful estimates of source parameters***
  - *Estimate the value of source parameters*
  - *Figure of merit is uncertainty in parameter estimation, e.g., luminosity distance, masses, spin parameters, sky location, etc.*
  - *Observation is really the desired metric of scientific capability.*

Detection with LISA



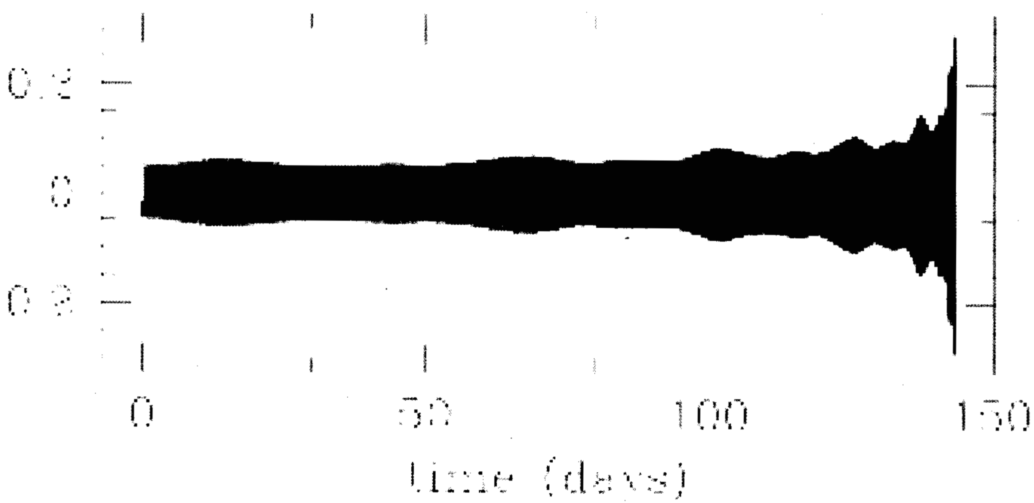
Observation with LISA

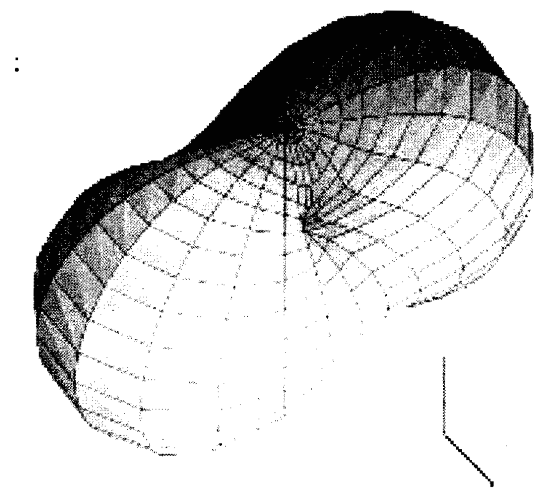
- ***The range of sources is large.***
- ***Our science objectives are diverse.***
- ***How do we know that the instrument we've designed can satisfy our science objectives?***
- ***What other science objectives might it satisfy? What's the discovery space?***
- ***Here's the process:***
  - *Write down the objectives*
  - *Formulate the desired science observations (which we think that we can fulfill) which will satisfy the objectives, with requirements on the estimation of source parameters*
  - *Calculate a performance model of the instrument*
  - *Analyze whether that instrument can perform those observations.*
- ***And accommodate the possible range of source/detector interactions.***

The Unknown Source

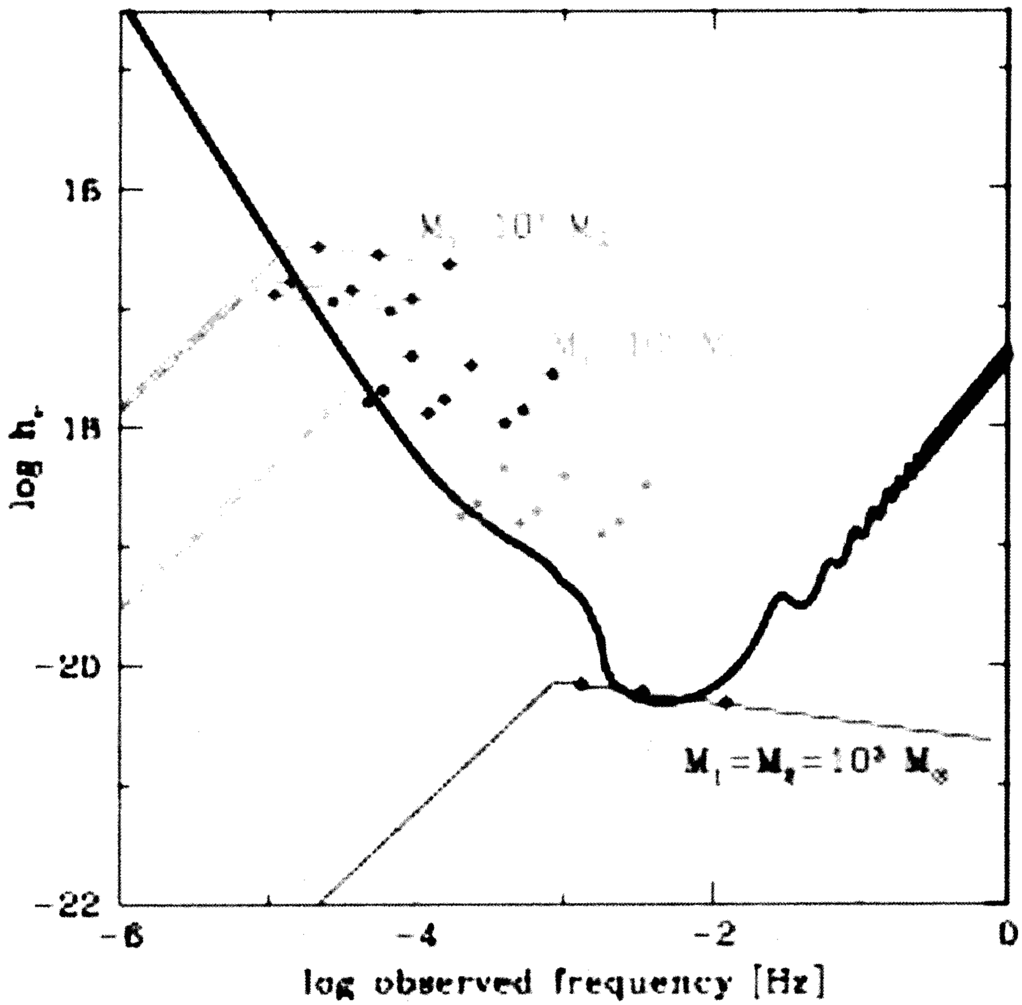
- ***Waveform depends on 17 source parameters***
  - *Intrinsic parameters of the source: m_1, m_2, $\mathbf{s_1}$, $\mathbf{s_2}$, eccentricity, separation, orbital phase. [Chirp mass and reduced mass.]*
  - *Extrinsic parameters of the observer: luminosity distance, sky position, orientation of the source orbit on the sky, orientation of the detector, merger time, orbital phase of the detector.*

$$h \quad \frac{[G(1+z)\mathcal{M}/c^2]^{5/3}\,[\pi f(t)/c]^{2/3}}{D_L}\mathcal{F}(\text{``angles''})\cos[\Phi(t)]$$





Evolution of the characteristic strain signal

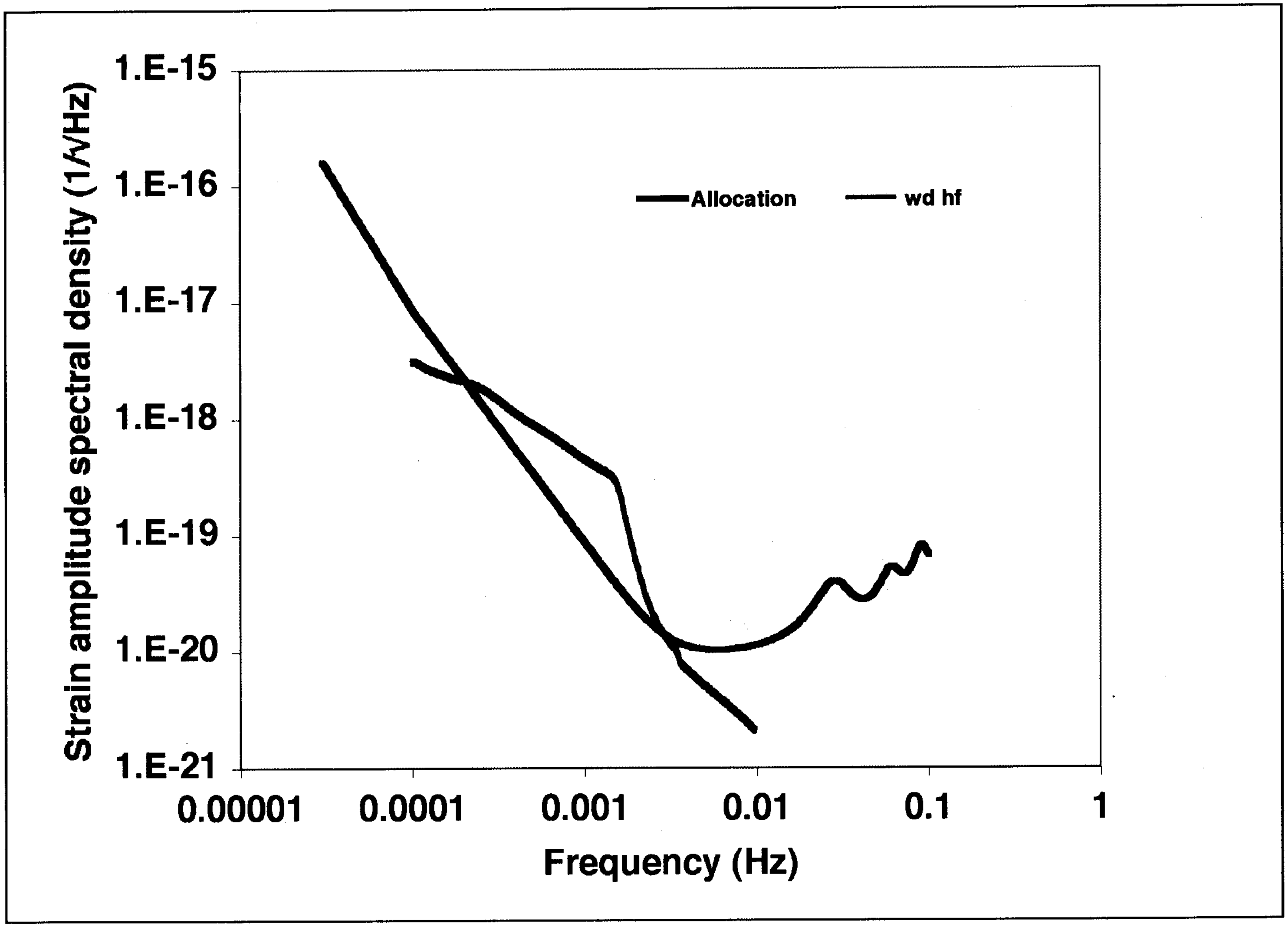


Characteristic strain for various masses ($10^{5\text{-}7}$, 10^3 $M_\odot$) and redshifts (1,3,5,7), 1 yr, 1 mo and 1 day. (Sesana, Haardt, Madau and Volonteri, 2004)

Instrument Assumptions

- ***Starting frequency for integration***
  - *Lower edge of detector sensitivity (3×10^{-5} or 1×10^{-4} Hz)*
  - *Time before coalescence*
- ***Number of interferometers***
  - *6 links (all 3 arms)*
  - *4 links (1 equivalent Michelson)*

Instrument Sensitivity Model



Science Objectives and Investigations

- *LISA science spans seven science objectives*

Understand the formation of massive black holes	*Confront General Relativity with observations*
Trace the growth and merger history of massive black and their host galaxies	*Probe new physics and cosmology with gravitational waves*
Explore stellar populations and dynamics in galactic nuclei	*Search for unforeseen sources of gravitational waves*
Survey compact stellar-mass binaries and study the structure of the Galaxy	

- ***Each science objective has one or more science investigations by which it will be achieved. For example,***

Trace the growth and merger history of massive black holes and their host galaxies	*Determine the relative importance of different black hole growth mechanisms as a function of redshift*
	Determine the merger history of $1x10^4$ to $3x10^5$ $M_\odot$ black holes from the dawn of galaxies (z~20) to the era of the earliest known quasars (z~6)
	Determine the merger history of $3x10^5$ to $1x10^7$ $M_\odot$ black holes at later epochs (z<6)

Science Requirements

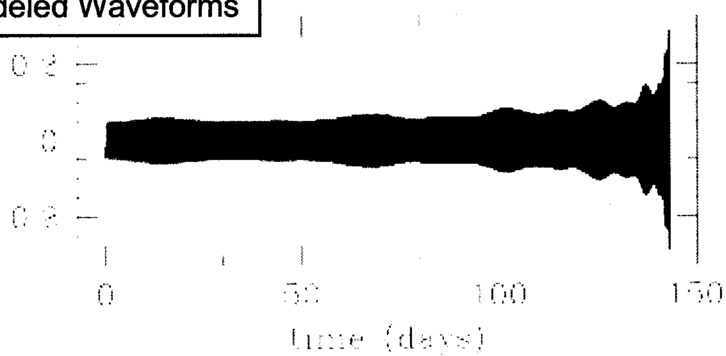
Science Investigation

4.2.1 Determine the relative importance of different black hole growth mechanisms as a function of redshift

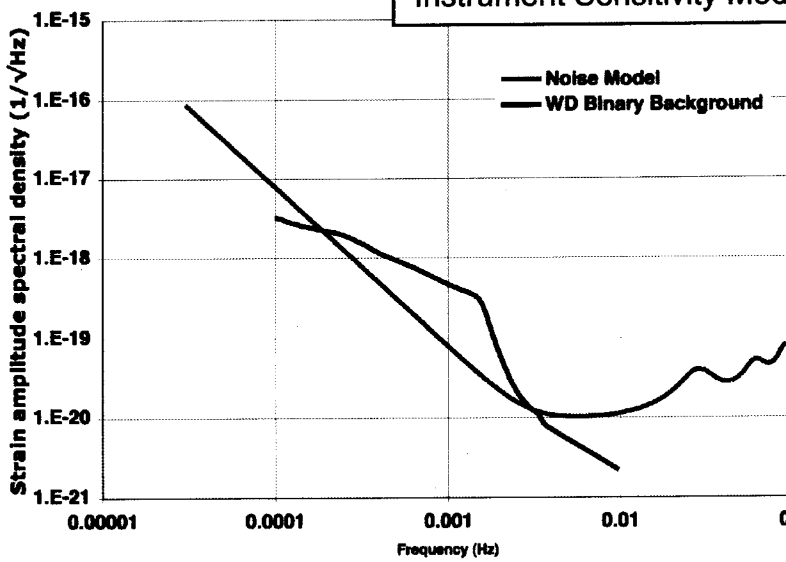
Observation Requirement

OR2.1: LISA shall have the capability to detect massive black hole binary mergers, with the larger mass in the range $3x10^4\ M_\odot < M_1 < 3x10^5\ M_\odot$, and a smaller mass in the range $10^3\ M_\odot < M_2 < 10^4\ M_\odot$, at $z = 10$, with fractional parameter uncertainties of 25% for luminosity distance, 10% for mass and 10% for spin parameter at maximal spin. LISA shall maintain this detection capability for five years to increase the number of observed events.

Modeled Waveforms



Instrument Sensitivity Model



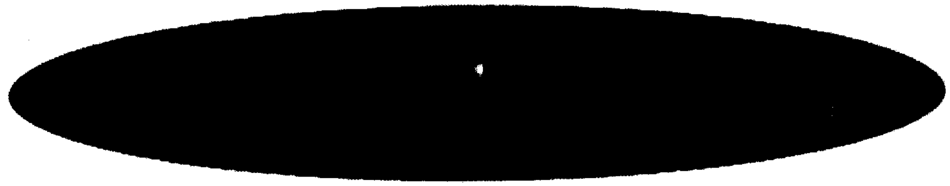
Parameter Uncertainties

M_1	M_2	D_L Uncertainty	Spin Uncertainty	SNR
1.00E+04	3.00E+02	31.90%	0.012	10.80
	1.00E+03	34.10%	0.029	18.50
	3.00E+03	43.20%	0.070	30.90
	1.00E+04	41.10%	0.115	47.90
3.00E+04	3.00E+02	28.50%	0.005	14.90
	1.00E+03	26.80%	0.008	26.40
	3.00E+03	25.00%	0.016	45.30
	1.00E+04	24.20%	0.041	79.50
1.00E+05	3.00E+02	31.70%	0.005	14.60
	1.00E+03	23.30%	0.006	27.80
	3.00E+03	20.20%	0.008	46.00
	1.00E+04	19.30%	0.020	75.00
3.00E+05	3.00E+03	22.50%	0.016	10.20

Doing the Calculations

- ***A work in progress***
- ***Two methods***
  - *LISA Calculator*
    - *Handy dandy web-based interface, with user selectable parameters*
    - *Doesn't include spin effects*
    - *Calculates one source scenario*
    - *Problems with large (>100) mass ratios*
  - *Ryan Lang & Scott Hughes, does include spin effects*
    - *Laborious*
    - *Includes spin effects*
    - *Monte Carlo approach to spin, sky orientation, merger time*
    - *Problems with equal mass*
- ***Not included***
  - *Merger and ring down*
  - *Comprehensive exploration*

LISA Calculator



Within the time of observation chosen, the binary system would be too relativistic for the 2PN assumptions used by the calculator. The observation time was changed to 0.999982 years.

The source is chirping. The uncertainties in the set of nine parameters are:

Parameter	Parameter Value	Parameter Uncertainty
Latitude (θ)	-20°	26.1531°
Azimuthal Location (φ)	305°	45.741°
Coalescence Time (t_c)	1 year	9.88254e-006 year
Luminosity Distance (D_L)	1.06255e+008 kpc	6.55608e+007 kpc
Inclination (ι)	90°	17.182°
Polarization Angle (ψ)	0°	13.1357°
Chirp Mass (M_c)	4607.7 M_{sun}	0.320249 M_{sun}
Reduced Mass (μ)	2307.69 M_{sun}	5.97818 M_{sun}
Orbital Phase (φ_o)	0°	171.618°
Sky Angle (sqrt($\Delta\Omega$))	---	59.2°

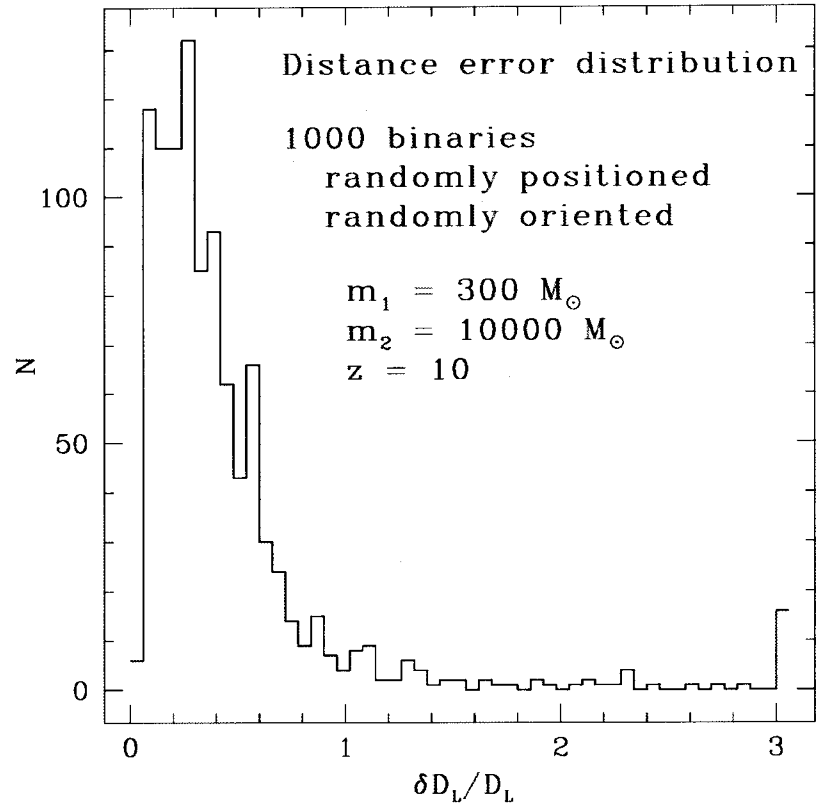
Signal to noise ratio of the source: 28.6833

The following information is included for reference:

Observation Time: 0.999982 years
Initial Frequency: 0.268265 mHz
Final Frequency: 16.0827 mHz
Frequency change (in units of f_m = 1/(year)): 499075
Time to coalescence: 1 years
Luminosity Distance: 1.06255e+008 kpc
Redshift: 10

Lang & Hughes

- ***Lang & Hughes calculation***
  - *Full 2 PN waveform simulation*
  - *Sky and polarization averaged*
  - *1 and 2 interferometers*
  - *Monte Carlo spins*
  - *Median performance*



Parameter Uncertainties

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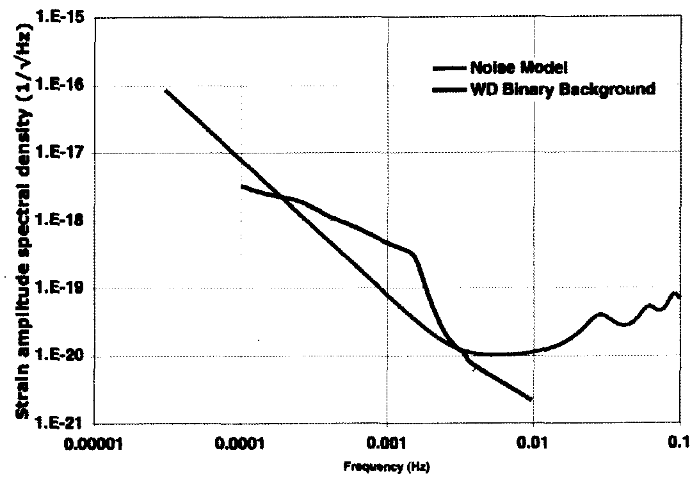
	1.00E+04	24.20%	0.041	79.50
1.00E+05	3.00E+02	31.70%	0.005	14.60
	1.00E+03	23.30%	0.006	27.80
	3.00E+03	20.20%	0.008	46.00
	1.00E+04	19.30%	0.020	75.00
3.00E+05	3.00E+03	22.50%	0.016	10.20

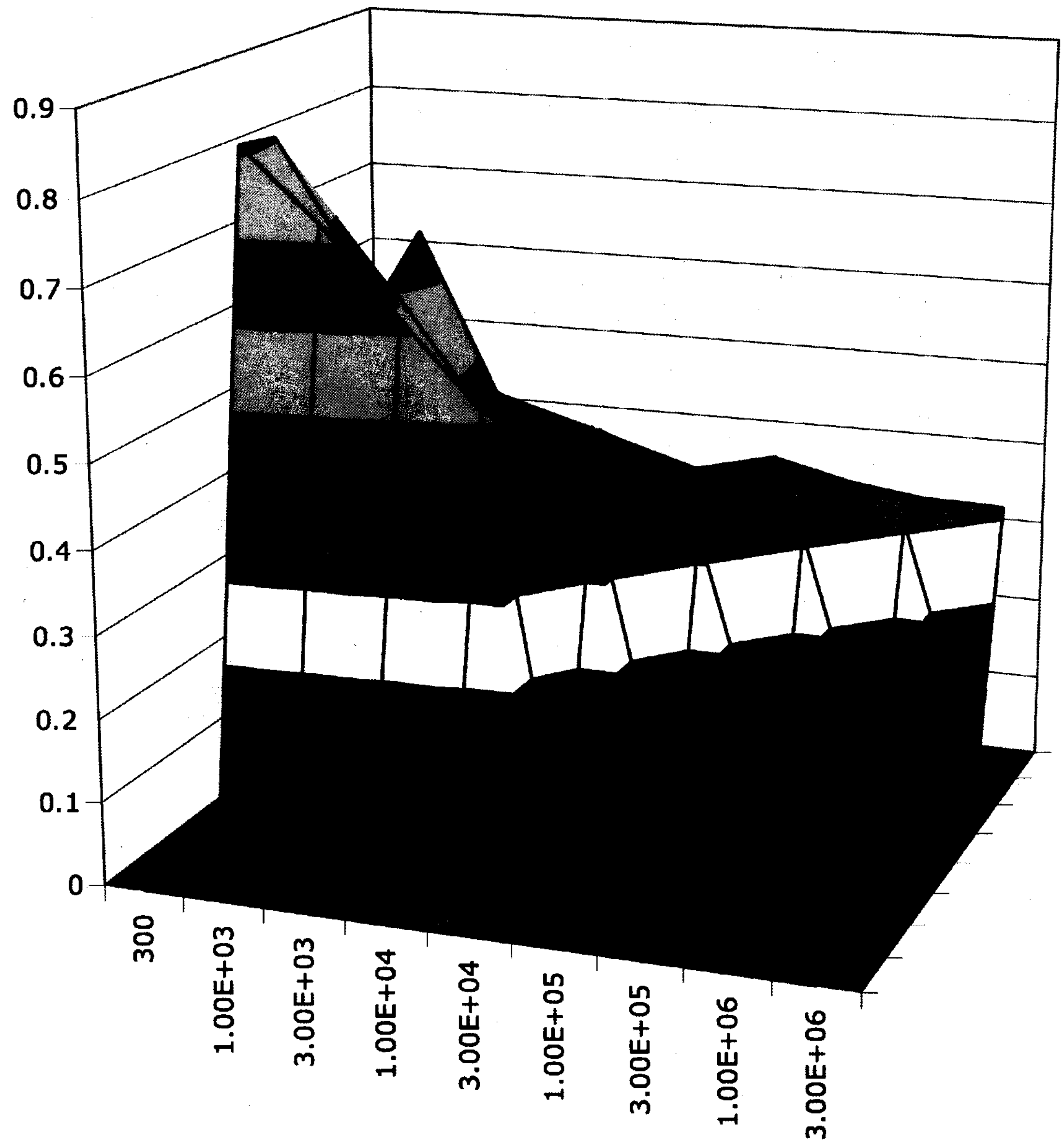
Performance, using the LISA Calculator

M1	M2	D_L	D_L Uncertainty	1 IFO D_L Uncertainty	1 IFO SNR	Initial Frequency (mHz)	Final Frequency (mHz)
1.00E+04	3.00E+02	1.06E+08	8.62E+07	81.09%	9.7	0.617	24.344
	1.00E+03	1.06E+08	7.69E+07	72.38%	17.1	0.396	20.713
	3.00E+03	1.06E+08	6.56E+07	61.70%	28.7	0.268	16.083
	1.00E+04	1.06E+08	5.48E+07	51.60%	48.0	0.180	10.797
3.00E+04	3.00E+02	1.06E+08	8.54E+07	80.33%	12.2	0.467	8.816
	1.00E+03	1.06E+08	6.99E+07	65.74%	21.8	0.299	8.501
	3.00E+03	1.06E+08	5.78E+07	54.41%	37.5	0.200	7.766
	1.00E+04	*1.06E+08*	4.78E+07	44.95%	62.0	0.130	6.219
	3.00E+04	1.06E+08	4.18E+07	39.34%	75.5	[illegible]	4.267
1.00E+05	3.00E+02	1.06E+08			4.8	0.342	2.681
	1.00E+03	1.06E+08	6.50E+07	61.16%	8.7	0.219	2.663
	3.00E+03	1.06E+08	5.17E+07	48.64%	14.5	0.146	2.613
	1.00E+04	1.06E+08	4.28E+07	40.32%	22.9	[illegible]	2.454
	3.00E+04	1.06E+08	3.82E+07	35.91%	28.8	[illegible]	2.096
	1.00E+05	1.06E+08	3.54E+07	33.31%	30.7	[illegible]	1.396
3.00E+05	1.00E+03	1.06E+08	6.99E+07	65.74%	21.8	0.299	8.501
	3.00E+03	1.06E+08	4.90E+07	46.16%	5.6	0.110	0.889
	1.00E+04	1.06E+08	4.06E+07	38.23%	10.5	[illegible]	0.874
	3.00E+04	1.06E+08	3.67E+07	34.50%	17.7	[illegible]	0.831
	1.00E+05	1.06E+08	3.44E+07	32.35%	28.4	[illegible]	0.704
	3.00E+05	1.06E+08	3.34E+07	31.44%	35.0	[redacted]	0.479
1.00E+06	1.00E+03	1.06E+08		0.00%	1.3	0.114	0.269
	3.00E+03	*1.06E+08*		*0.00%*	2.9	[illegible]	*0.268*
	1.00E+04	1.06E+08	4.21E+07	39.62%	6.1	[illegible]	0.267
	3.00E+04	1.06E+08	3.72E+07	35.05%	10.8	[illegible]	0.263
	1.00E+05	1.06E+08	3.47E+07	32.65%	18.8	[redacted]	0.251
	3.00E+05	1.06E+08	3.35E+07	31.49%	28.1	[redacted]	0.218
	1.00E+06	1.06E+08	3.30E+07	31.02%	31.6	[redacted]	[illegible]
3.00E+06	1.00E+03	1.06E+08		0.00%	0.3	[illegible]	[illegible]
	3.00E+03	1.06E+08		0.00%	0.7	[illegible]	[illegible]
	1.00E+04	1.06E+08		0.00%	2.0	[illegible]	[illegible]
	3.00E+04	1.06E+08	3.50E+07	32.97%	14.3	[redacted]	[illegible]
	1.00E+05	1.06E+08	3.75E+07	35.31%	8.6	[redacted]	[illegible]
	3.00E+05	1.06E+08	3.50E+07	32.97%	14.3	[redacted]	[illegible]
	1.00E+06	1.06E+08	3.36E+07	31.65%	21.1	[redacted]	[illegible]
	3.00E+06	1.06E+08	3.33E+07	31.39%	21.2	[redacted]	[illegible]



Calculator D_L Uncertainty z=10	100	300	1.00E+03	3.00E+03	1.00E+04	3.00E+04	1.00E+05	3.00E+05	1.00E+06	3.00E+06	1.00E+07
100										[illegible]	
300					81.09%	80.33%				[illegible]	
1.00E+03					72.38%	65.74%	61.16%	65.74%		[illegible]	
3.00E+03					61.70%	54.41%	48.64%	46.16%	[illegible]	[illegible]	
1.00E+04					51.60%	44.95%	40.32%	[illegible]	[illegible]	[illegible]	
3.00E+04						[illegible]	[illegible]	[illegible]	[illegible]	[redacted]	
1.00E+05							[illegible]	[illegible]	[illegible]	[redacted]	
3.00E+05								[illegible]	[illegible]	[redacted]	
1.00E+06									[illegible]	[redacted]	
3.00E+06										[redacted]	
1.00E+07											

Distance Uncertainty



Lessons learned (so far)

- *Luminosity distance is the most fragile parameter.*
- *Distance uncertainty doesn't track SNR!*
- *Mass is a phase measurement*
- *Distance is an amplitude measurement.*
- *Inclusion of spins gives factors of 1-6 improvement.*
- *Inclusion of merger and ring-down phases may give an additional factor of 3.*
- *We haven't included the hierarchical merger trees yet. Including spins.*

Summary

- ***Converted the science requirements from SNR based detection to parameter estimates from observations***
- ***The adequacy of the instrument performance can only be verified by the forward calculation showing that the science requirements can be met. The instrument performance cannot be directly calculated from the science requirements.***
- ***The performance of the instrument and the discovery space can be represented by "contours" of parameter estimation in an m_1/m_2 coordinate system.***